$$p[n] = \begin{cases} 1, & n = 0, 1, 2, \cdots, q-1 \\ 0, & \text{all other } n \end{cases}$$

$$= u[n] - u[n-q]$$

$$u[n] \leftrightarrow \frac{z}{z-1} \quad ; \quad u[n-q] \leftrightarrow \frac{z}{z-1} z^{-q}$$

$$p[n] \leftrightarrow \frac{z}{z-1} - \frac{z}{z-1} z^{-q}$$

$$\leftrightarrow \frac{z}{z-1}\left(1 - z^{-q}\right) = \frac{z}{z-1}\left(1 - \frac{1}{z^q}\right)$$

$$\leftrightarrow \frac{z(1 - z^{-q})}{z-1}$$

Ex

$$x[n] = na^n u[n]$$

$$a^n u[n] \leftrightarrow \frac{z}{z-a}$$

$$X(z) = -z \frac{d}{dz}\left(\frac{z}{z-a}\right)$$

$$= -z \frac{(z-a)(1) - (z)(1)}{(z-a)^2} = -z \frac{z-a-z}{(z-a)^2}$$

$$= \frac{az}{(z-a)^2}$$

$$na^n u[n] \leftrightarrow \frac{az}{(z-a)^2}$$

Ex $n^2 a^n u[n] \leftrightarrow ?$

$a^n u[n] \leftrightarrow \frac{z}{z-a}$

Apply $n^2 x[n] \leftrightarrow z\frac{d}{dz}X(z) + z^2\frac{d^2}{dz^2}X(z)$

OR

$n a^n u[n] \leftrightarrow \frac{az}{(z-a)^2}$

Apply $nx[n] \leftrightarrow -z\frac{d}{dz}X(z)$

Ex $P[n] = u[n] - u[n-q]$

$P[n] \leftrightarrow \frac{z(1-z^{-q})}{z-1}$

$a^n P[n] \leftrightarrow \frac{(z/a)(1-(z/a)^{-q})}{(z/a)-1} \times \frac{a}{a}$

$\leftrightarrow \frac{z(1-(z/a)^{-q})}{z-a}$

$\leftrightarrow \frac{z(1-a^q z^{-q})}{z-a}$

Ex

$x[n] = u[n]\cos(\Omega n)$

$(\cos \Omega n)x[n] \leftrightarrow \frac{1}{2}[X(e^{j\Omega}z) + X(e^{-j\Omega}z)]$

$u[n] \leftrightarrow \frac{z}{z-1}$

$$u[n]\cos(\Omega n) \leftrightarrow \frac{1}{2}\left[\frac{z e^{j\Omega}}{z e^{j\Omega} - 1} + \frac{z e^{-j\Omega}}{z e^{-j\Omega} - 1}\right]$$

$$\leftrightarrow \frac{1}{2}\left[\frac{z e^{j\Omega}(z e^{-j\Omega} - 1) + z e^{-j\Omega}(z e^{j\Omega} - 1)}{(z e^{j\Omega} - 1)(z e^{-j\Omega} - 1)}\right]$$

$$\leftrightarrow \frac{1}{2}\left[\frac{z^2 - z e^{j\Omega} + z^2 - z e^{-j\Omega}}{z^2 - z(e^{j\Omega} + e^{-j\Omega}) + 1}\right]$$

$$\leftrightarrow \frac{1}{2}\left[\frac{2z^2 - 2z\left(\frac{e^{j\Omega} + e^{-j\Omega}}{2}\right)}{z^2 - 2z\left(\frac{e^{j\Omega} + e^{-j\Omega}}{2}\right) + 1}\right]$$

$$\leftrightarrow \frac{z^2 - z\cos(\Omega)}{z^2 - 2z\cos(\Omega) + 1}$$

$$v[n] = \sum_{i=0}^{n} u[i] = (n+1)u[n]$$

$$u[n] \leftrightarrow \frac{z}{z-1}$$

$$\sum_{i=0}^{n} x[i] \leftrightarrow \frac{z}{z-1} X(z)$$

$$\therefore V(z) = \frac{z}{z-1} X(z)$$

$$= \frac{z}{z-1} \cdot \frac{z}{z-1} = \frac{z^2}{(z-1)^2}$$

Find the initial & final values of

$$X(z) = \frac{z-1}{z^2+3}$$

$$x[0] = \lim_{z \to \infty} X(z) = \lim_{z \to \infty} \left(\frac{z-1}{z^2+3} \right) = 0$$

$$x[\infty] = \lim_{n \to \infty} x[n] = \lim_{z \to 1} (z-1) X(z)$$

$$= \lim_{z \to 1} (z-1) \left(\frac{z-1}{z^2+3} \right)$$

$$= \lim_{z \to 1} \frac{(z-1)^2}{z^2+3} = 0$$

EX

$$X(z) = \frac{3z^2 - 2z + 4}{z^3 - 2z^2 + 1.5z - 0.5}$$

The poles are $1;\ 0.5+j0.5;\ 0.5-j0.5$

$$X(z) = \frac{3z^2 - 2z + 4}{(z-1)(z-0.5-j0.5)(z-0.5+j0.5)}$$

$$(z-1)X(z) = \frac{3z^2 - 2z + 4}{(z-0.5-j0.5)(z-0.5+j0.5)}$$

EX

$X(z) = \frac{z}{z-1}$, find $x[n]$

$x[n] = u[n]$

or

$$1 + z^{-1} + z^{-2} + \cdots \leftarrow$$

$$z-1 \;\overline{)\; z}$$

$$z - 1$$

$$1$$

$$1 - z^{-1}$$

$$z^{-1}$$

$$z^{-1} - z^{-2}$$

$$z^{-2}$$

$$\vdots$$

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$$

$$U(z) = \sum_{n=0}^{\infty} u[n] z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots$$